Determination of arch form using polynomial spline functions

ABSTRACT: To establish a suitable upper and lower arch form based o function for subjects of Chennai population. Majority of the commercially preformed arch forms used in pre-adjusted system has been found to be not u applicable. This could be because of racial and ethnic variation in the size a in human dental arch forms due to genetic and environmental factors. 90 arch forms used result in unwanted expansion in the canine region. This v the post treatment stability and will lead to relapse. Thus it is important th form should be established pertaining to the population being treated. At various geometric forms studied spline function has been reported to be mor and reliable according to Begole. Spline is a mathematical model of physic used by draftsman to draw smooth curve.

Achievement of a stable functional and aesthetic arch form has long been one of the prime objectives of orthodontics. The search for an ideal arch form has been one of the most persistent but elusive tasks that orthodontic researchers have pursued. Researchers have been trying to define the arch form frequently based on the concept that the dental arch is symmetric in nature and can be represented by an Algebraic or Geometric formula.

Many studies have been conducted to predetermine a Mathematical and Geometric arch form on the basis of landmarks recorded on system of coordinates which include Parabola, Catenary curve, Conic sections and various polynomial functions. Of these conic sections the simplest plane curves after straight lines comprise rather a flexible family, which includes circles ellipses hyperbolas and seven parts of straight lines. Shape investigated includes the ellipse, parabola and catenary curve, as well as mathematical formula such as cubic spline, conic sections and polynomial functions. All the commercially produced arch forms are derived from one of this form.

The physical spline consists of a thin flexible strip of wood or plastic which is laid over graph paper and held in such a way that it is forced to pass through certain data points. The draftsman then traces along the spline to obtain the curve. These points are called knot points. in order to mathematically fit a spline to a set of data points a set of n knot points must be selected based on the restriction that $X_1 < X_2 < \ldots . X_n$. A spline polynomial is defined between each successive pairs of knots and subsequently the curve is smoothened at the knots resulting in a spline which passes precisely through the values chosen as knots. Thus the fit of spline reside in the choice of knot points.

MATERIALS

The sample for the study consisted of 60 subjects with normal occlusion in the age group of 13- 18 years selected by screening 3350 children from 4 schools in Chennai, Out of the 60 selected subjects, 27 were males , 33 were females. The selected subjects belonged to the local population of Chennai.

All the subjects had

1. Angles Class I molar relation and Class I canine relation.
2. Normal overjet and overbi mm
3. Fully erupted normal compl all permanent teeth.
4. Well- aligned upper and lo forms(within acceptable
5. No dental caries or rest involving the inter proximal

Method: Data collection, Dat and evaluation, Arch modeli polynomial spline function, Co of selected preformed arches respective arch models.

SOFTWARE USED:

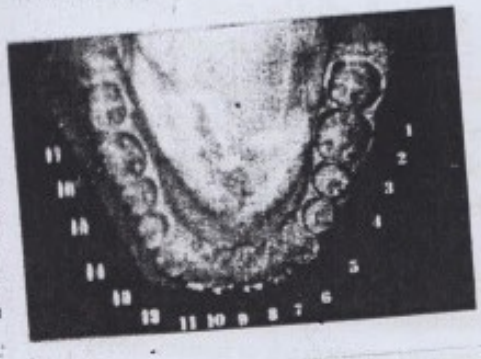
Microsoft Excel, Table Curve Math Cad and Auto Cad

DATA COLLECTION.

A single person in daylight exar selected subjects. Alginate imp were taken for both upper a arches. Casts were prepared fr

impressions immediately to avoid dimensional changes. Both upper and lower models for each subject were carved and labeled.

Fig 2



midline were marked on both upper and lower models to establish X and Y coordinates for construction of each arch form by spline functions. Mid- line marking in the maxillary cast was done by selecting a point posterior to the incisor pappila and another point in the posterior part of the cast on the mid palatal raphe distal to this point. In lower arch the midline was drawn between to selected points, a point marked between the two central incisors inter dental areas and another p the posterior part transferred fr upper model keeping them in occ All the 17 points were marked ea with a permanent marker.

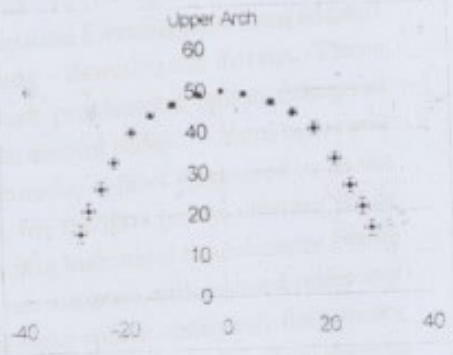
The points selected for marking w follows

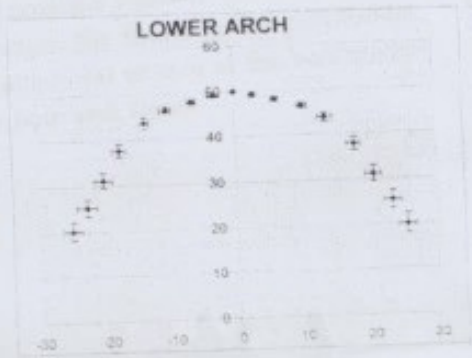
1. The incisal mid point of the incisor (point 8& 10)
2. Mesio incisal and disto incisal of lateral incisor(point 6, 7,11
3. Buccal cusp tip of canines(p 13)
4. Buccal cusp tip of premolar 3,4 , 14&15)
5. Disto buccal and mesiobuccal of first molar(points1,2,

Out of the 17 point selected to generate the spline curve disto buccal and mesio buccal cusp tips of the first permanent molar (point 1,2, 16&17) buccal cusp tip of canine (point 5, 13), mid point between the mesio incisal corners of the central incisor (point 9) were taken as knot points. Rest of the points are known as floating points, which add the flexibility to the curve construction.

These points had been marked on the dental casts before digitization to ensure proper location of the landmark. The marked models were photocopied with the occlusal plane resting on the glass surface with a millimeter ruler placed in the field to correct for magnification errors related to the copying process. The cast was placed in the center of the camera lens of the copying machine to minimize distortion in the copied image of the cast. The data points from the photocopies of the models were transferred to separate graph sheets (unit measurement of 1mm) with the help of a template. Separate X and Y axes with length of 60mm each were drawn in the graph paper. From the mid point of X axis a perpendicular line of 50 mm length was also drawn to correspond to the midline of each cast. The template was prepared by photo copying a graph sheet with the above line drawn to a transparent OHP sheet in 1:1 measurement to get actual size. The template was placed over the photocopies of individual models with midline of the X axis coinciding with the midline of the model. The distances of the 17 selected data points were measured to give X and Y coordinates and plotted on to individual graph sheet. For the X coordinates left side of the midline was considered as –ve and the right side as +ve. The measure-ments were also numerically entered in the graph sheet for further use. The data obtained from the 60 subjects were entered in Microsoft Excel.

The mean, standard deviation, correlation coefficient(r) and r^2 values between X and Y coordinates of all the 60 subjects were calculated for each of the 17 data points separately for the upper and lower arches using Microsoft Excel. The figure shows the Spatial distribution of the individual inflection points.



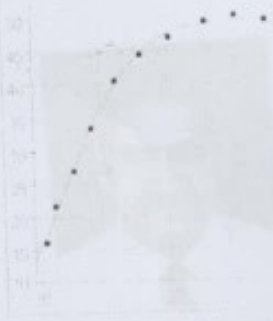


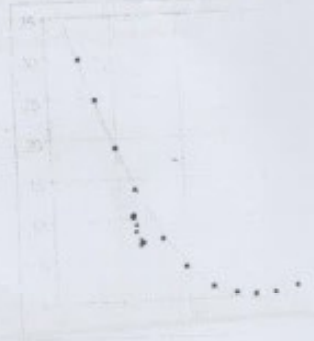
The mean value obtained was used for modeling the arch using various splines . The following three different polynomial splines were selected for modeling.

1. $Y=Sqrt(a+bX+cX^2)$ (A second order non linear model)
2. $Y = a+bX+cX^2+dX^3$ (A third order cubic spline)
3. $Y= a+bX+cX^2+dX^3+eX^4$ (A fourth order polynomial spline

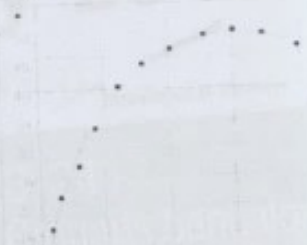
The mean squared err was chosen for selec appropriate model. The estimated using Math C Table curve software.

SECOND ORDE FUNCTION



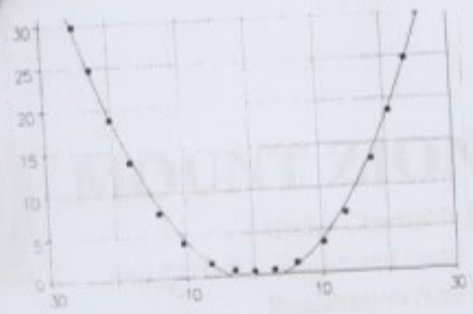


III ORDER CUBIC



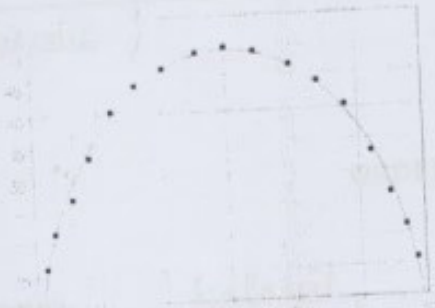
Comparison of cost functions

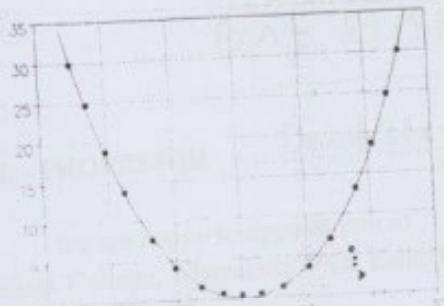
Model	Cost Function		Mode
	Lower arch	Upper Arch	
Second Order Spline	1.857	5.146	II
Third Order Spline	6.238	8.342	III
Fourth Order Spline	1.347	3.867	I



linear, third order cubic spline and fourth order spline. Table above shows the comparison of the cost function for three different splines used for modeling. Based on the cost function the fourth order polynomial function was chosen as the best model for both arch forms.

IV ODER POLYNOMIAL SPLINE





The size and shape of human dental arch have been studied over a century by various investigators. Recently mathematical formulas have been adapted to study dental arch forms. Three different polynomial spline functions namely second order third order and fourth order splines were used to fit the curve for the data points chosen. While analyzing individual model curve fitting was not accurate with second order and third order splines uniformly for anterior and posterior segment. Based on the cost function the fourth order polynomial function was chosen as the best model for both arch forms.

Analysis of change in arch form w
expansion AM.J.ORTHOD.199
No.3: 307-315:
Ellen .a.begole and RAYMOND
A new method for analyzing chang
arch form: AM J Orthod Dentofac
April 1998: Vol 113:394-401
C.L.BEAVELLE, The shape of d
AM J Orthod 1975: Vol 67: No.
MCLAUGHLIN, BENNETT &
MBT arch form and arch wire se

Dr. Kunjumon Zachariah MDS
Dept. of Orthodontics Royal Dent
Chalisbery, Palghat, Kerala

RESULTS

The study suggests that the tip of the mesio buccal cusp of the first molar canine and second premolar are more stable reference points especially on the left side of the arch.

For the lower arch the standard deviation of x and y value explained more variants in the first three and last three points, other points were less significant r values showed in all the points show a low positive relatjonship. Whereas r^2 value showed the points 1,2,3, 6&9 were explained nearly 12%- 17% of the total variation in the Y variables .Figure shows